Boxicity and related parameters

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Order & Geometry, Gułtowy Palace

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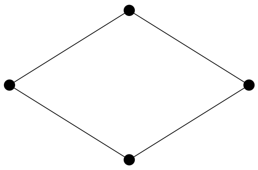

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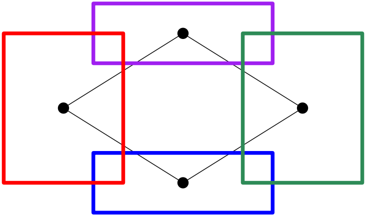

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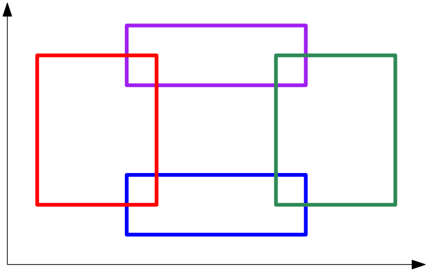

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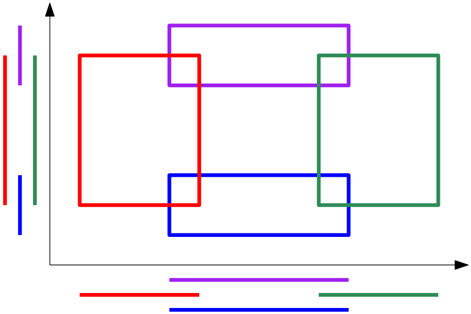

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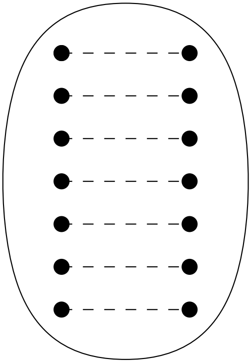
The boxicity of a graph $G = (V, E)$ is the smallest k for which there exist k interval graphs $G_i = (V, E_i)$, $1 \leq i \leq k$, such that $E = E_1 \cap \ldots \cap E_k$.

Graphs with large boxicity



K_n minus a perfect matching

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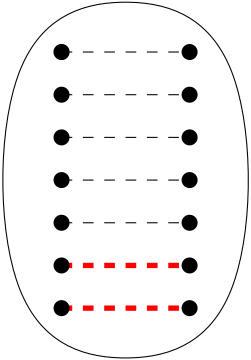

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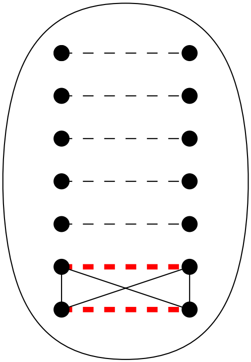

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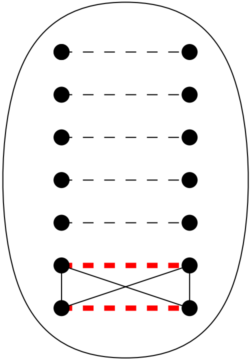

K_n minus a perfect matching

boxicity $n/2$

Boxicity and poset dimension

The dimension of a poset $\mathcal{P}$ is the minimum number of total orders realizing $\mathcal{P}$ (i.e. such that if $x <_{\mathcal{P}} y$ if and only if $x < y$ in all the total orders).

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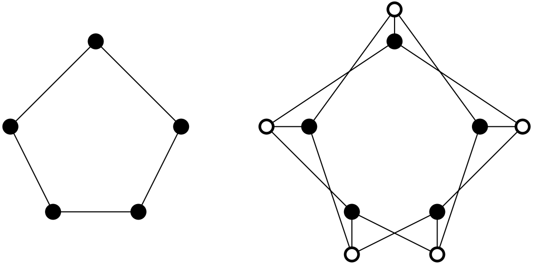

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Corollary (Adiga, Bhowmick, Chandran 2011)

If G is a graph and $\mathcal{P}$ is its extended double cover poset, then $\frac{1}{2}\dim(\mathcal{P}) - 2 \leq \text{box}(G) \leq 2\dim(\mathcal{P})$.

Dimension of the incidence poset

Incidence poset of G: the elements are the vertices and edges of G, with the inclusion relation.

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If G is a graph and $\mathcal{P}$ is its incidence poset, then $\mathrm{box}(G^*) \leq \dim(\mathcal{P}) \leq 2\,\mathrm{box}(G^*)$, where G^* denotes the 1-subdivision of G.

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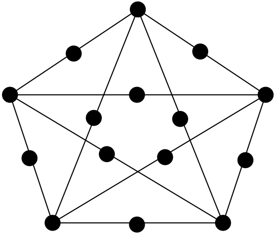
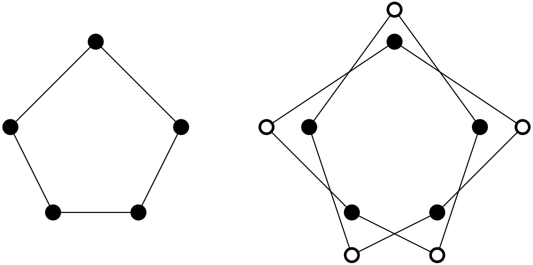


Subdivided K_n

boxicity $\Theta(\log \log n)$

Dimension of the adjacency poset

Adjacency poset of G (Felsner, Trotter 2000):



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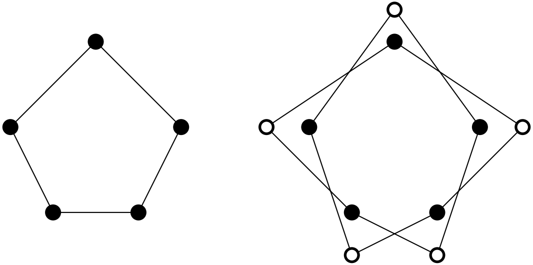

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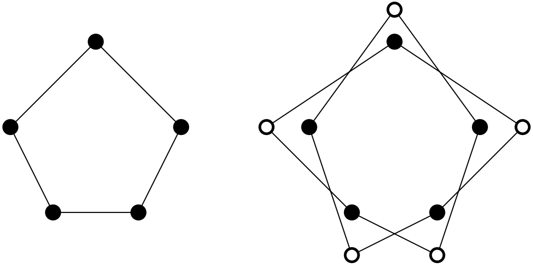

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Observation (E., Joret 2013)

If G is a graph and $\mathcal{P}$ is its adjacency poset, then $\dim(\mathcal{P}) \leq 2\,\mathrm{box}(G) + \chi(G) + 4$.

Separation dimension

Separation dimension of $G = (V, E)$ (Basavaraju, Chandran, Golumbic, Mathew, and Rajendraprasad 2014):
the minimum d such that there is a mapping $V \to \mathbb{R}^d$ such that for any two non-incident edges $uv, xy \in E$, some axis-parallel hyperplane separates $\{u, v\}$ from $\{x, y\}$.

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A fractional version was recently introduced (Loeb & West 2016) and (Alon 2016). It is always at most 3.

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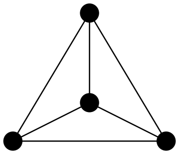

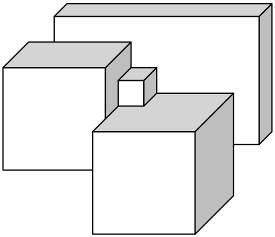

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Theorem (E. 2015)

Graphs with Euler genus g without non-contractible cycles of length at most $40 \cdot 2^g$ have boxicity at most 5.

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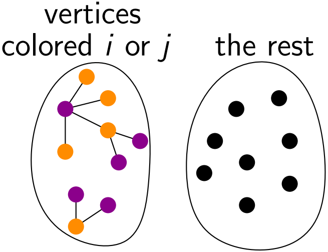

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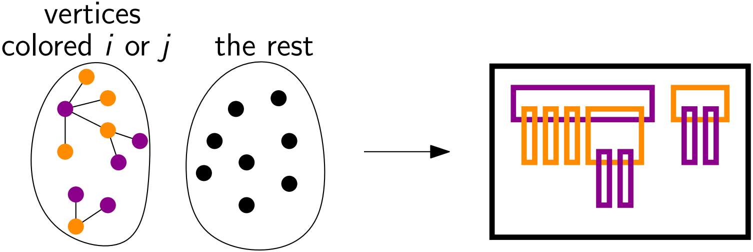

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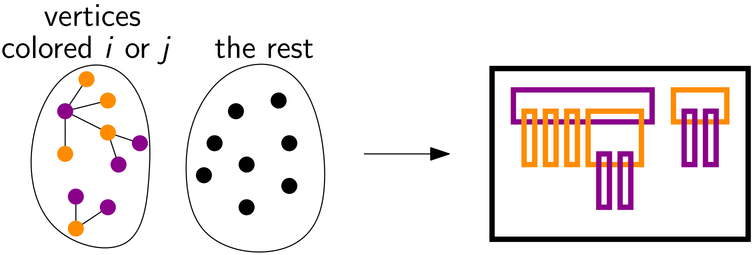

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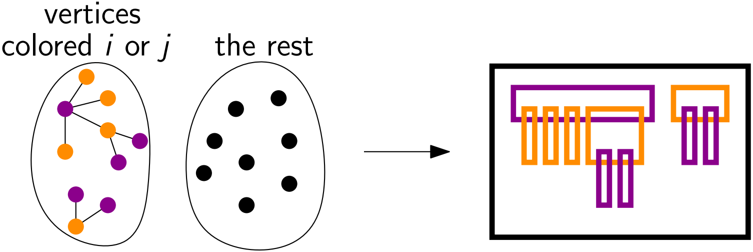

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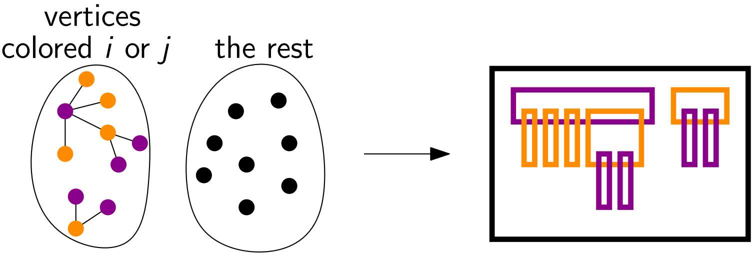

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$\Rightarrow \mathrm{box}(G) \leq k(k-1)$

Boxicity of graphs on surfaces

Theorem (Kawarabayashi, Thomassen 2012)

If a graph G has Euler genus g, then there is a set A of $O(g)$ vertices such that $G - A$ has an acyclic coloring with 7 colors.

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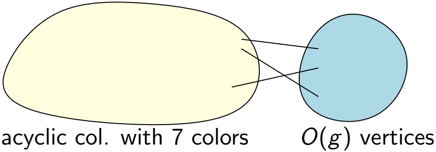

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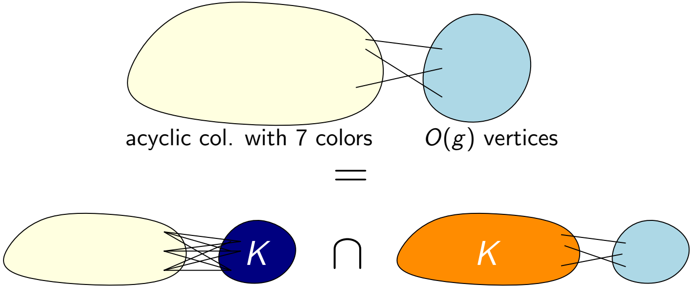

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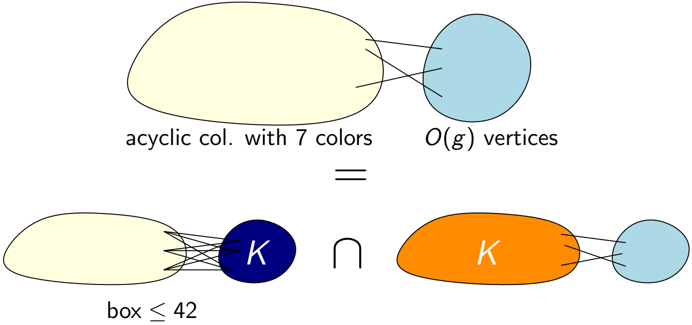

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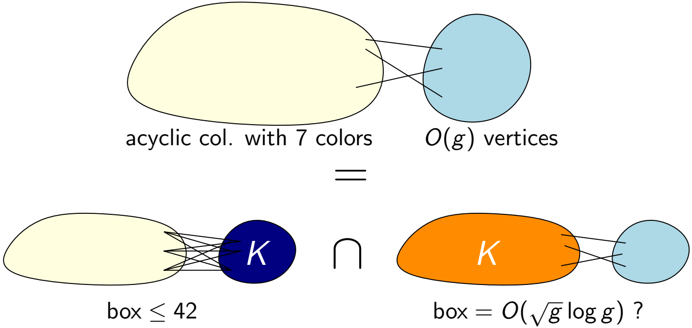

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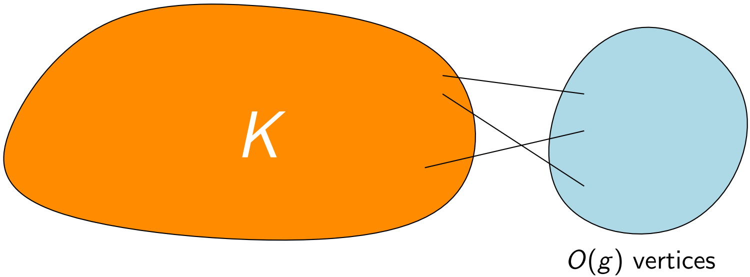

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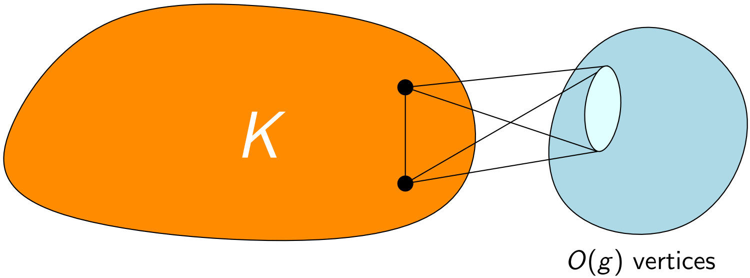

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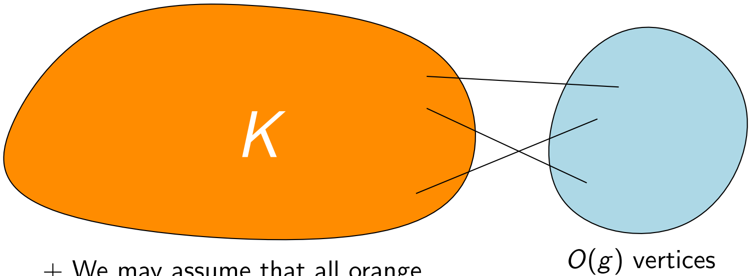

+ We may assume that all orange vertices have distinct blue neighborhoods

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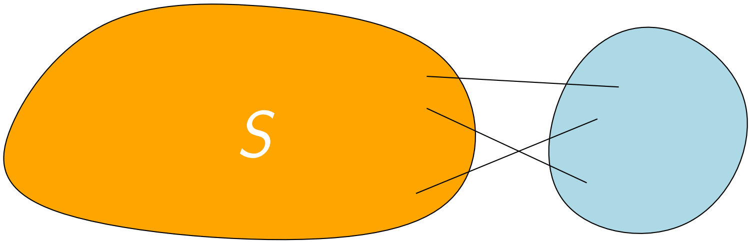

$O(g)$ vertices

+ We may assume that all orange vertices have distinct blue neighborhoods
+ stable set instead of clique

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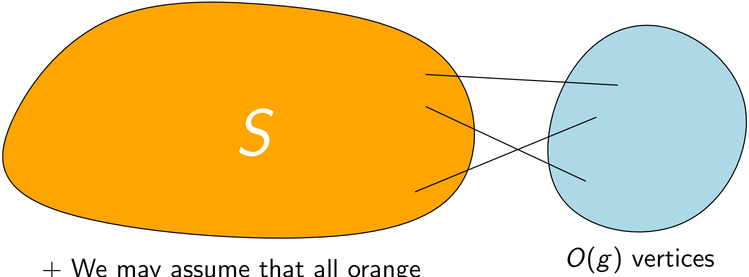

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$\Rightarrow$ the graph has $O(g^4)$ vertices

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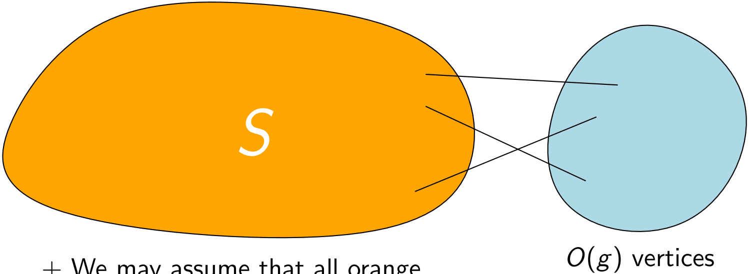

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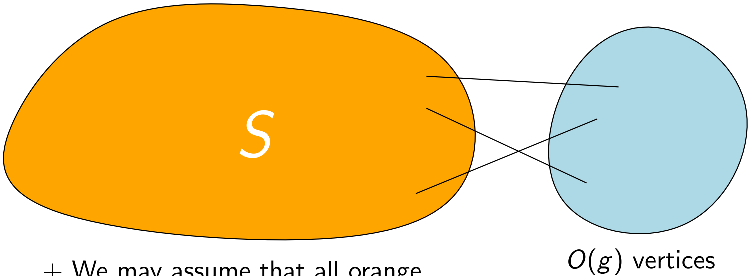

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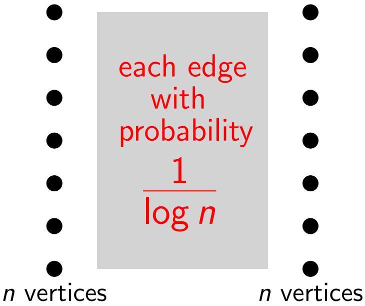
Theorem (Adiga, Chandran, Mathew 2014)

If a graph G with n vertices is k-degenerate, then $\mathrm{box}(G) = O(k \log n)$.

Lower bound

Consider the following random bipartite graph G_n:

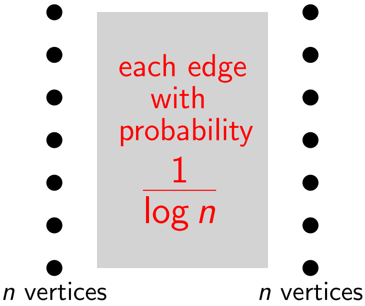


Lower bound

Consider the following random bipartite graph G_n:

with high probability,

G_n has at most $\frac{2n^2}{\log n}$ edges



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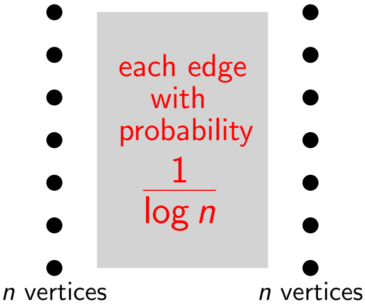

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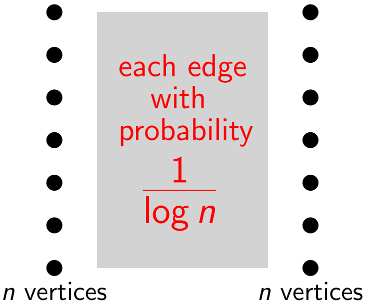

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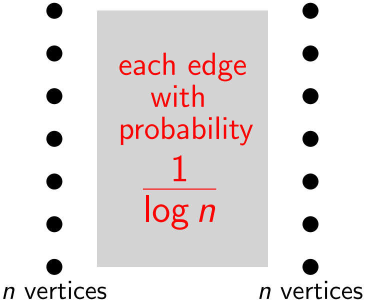

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Locally planar graphs

Theorem (E. 2015)

Graphs with genus g, without non-contractible cycles of length at most $40 \cdot 2^g$, have boxicity at most 5.

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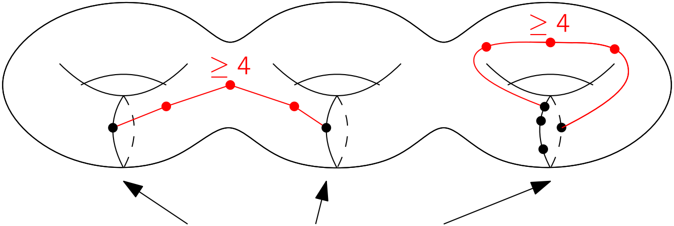

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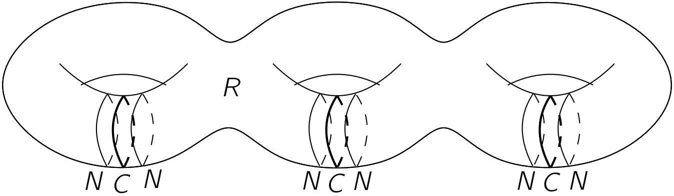

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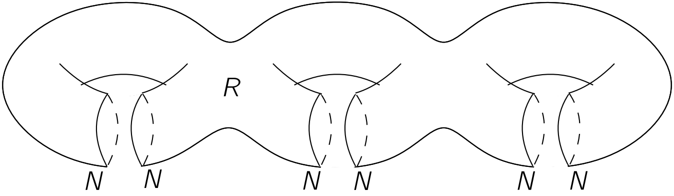

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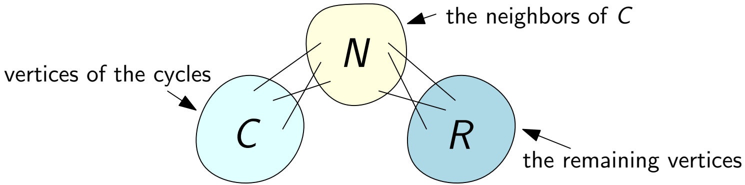

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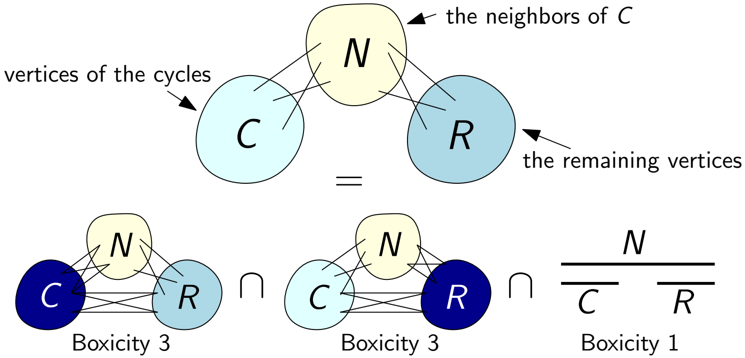

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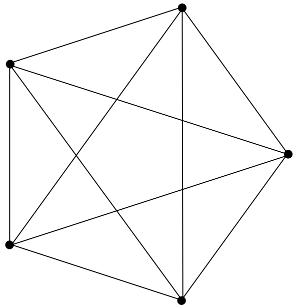

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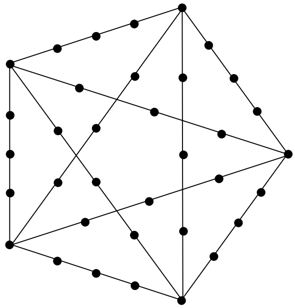

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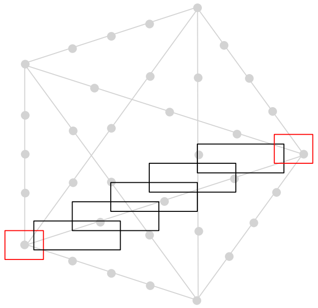

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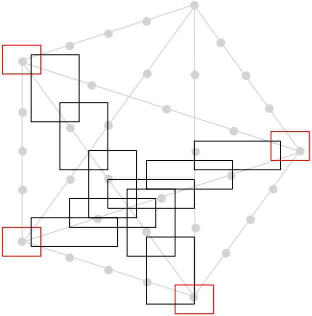

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There is a constant c such that any graph of Euler genus g and girth at least $c \log g$ has boxicity at most 3.

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Most of the questions remain interesting for the dimension of the adjacency poset and the separation dimension of graphs, instead of their boxicity.